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Specification of the 3GPP Confidentiality and Integrity Algorithms 128-EEA3 & 128-EIA3. Document 4: Design and Evaluation Report

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1.0	**11th August 2010**	**First public release**
1.1	**11th August 2010**	**A few typos corrected and text improved**
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Reference

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Foreword

This Report has been produced by the ETSI SAGE Task Force on the Design of the third LTE encryption and integrity protection algorithms 128-EEA3 and 128-EIA3 (SAGE TF 3GPP).

The work described in this report was undertaken in response to a request made by 3GPP.

1 Scope

This public report contains a detailed summary of the design and evaluation work performed during the development of the 3GPP Confidentiality and Integrity Algorithms 128-EEA3 and 128-EIA3. The report also includes summaries of evaluations made by independent external evaluators, and reflects modifications that amend the security flaws found by Bing Sun et al. [35] at the 2010 ZUC workshop, by Hongjun Wu [36] at the rump session of ASIACRYPT 2010, and by Thomas Fuhr et al. at the 2010 ZUC workshop/eprint [37].

2 References

For the purposes of this report, the following references apply:

[1] 3G TS 33.401 V 9.3.1 (2010-04) 3rd Generation Partnership Project; Technical Specification Group Services and System Aspects; 3GPP System Architecture Evolution (SAE); Security architecture (Release 9).

[2] ETSI/SAGE Specification. Specification of the 3GPP Confidentiality and Integrity Algorithms 128-EEA3 & 128-EIA3. Document 1: 128-EEA3 and 128-EIA3 Specification; Version: 1.5; Date: 4th January 2011.

[3] ETSI/SAGE Specification. Specification of the 3GPP Confidentiality and Integrity Algorithms 128-EEA3 & 128-EIA3. Document 2: ZUC Specification; Version: 1.5; Date: 4th January 2011.

Additional references to external documents are provided in Annex A.

3 Abbreviations

For the purposes of the present report, the following abbreviations apply:

AES	Advanced Encryption Standard
CK	Cipher Key
ETSI	European Telecommunications Standards Institute
GF(*q*)	The Galois field with *q* elements
3GPP	3rd Generation Partnership Project
EPS	Evolved Packet System (comprising LTE and SAE)
EEA	EPS Encryption Algorithm
EIA	EPS Integrity Algorithm
128-EEA3	Proposed third algorithm fulfilling the EEA requirement
128-EIA3	Proposed third algorithm fulfilling the EIA requirement
IBS	Input Bit Stream
IK	Integrity Key
IPR	Intellectual Property Rights
IV	Initialization Vector
LFSR	Linear Feedback Shift Register
LTE	Long Term Evolution (radio network)
MAC	Message Authentication Code
OBS	Output Bit Stream
OTP	One Time Pad
SA3	3GPP Systems and Architecture Group

SAE	System Architecture Evolution (core network)
SAGE	Security Algorithms Group of Experts
SAGE TF 3GPP	SAGE Task Force for the design of the standard 3GPP Confidentiality and Integrity Algorithms
XL	Extended Linearization

4 Structure of this report

The material presented in this report is organised in the subsequent sections, as follows:

- Section 5 provides background information on the third suite of 3GPP Confidentiality and Integrity Algorithms 128-EEA3 and 128-EIA3;
- Section 6 explains the different versions of the algorithms that have been evaluated and/or published;
- Section 7 provides a summary of the algorithm parameters;
- Section 8 gives a brief presentation of the actual designs;
- Section 8.3.2 provides information on the overall design rationale for ZUC;
- Section 10 provides information on the design and evaluation of ZUC components;
- Section 11 provides information on the overall rationale on 128-EEA3 and 128-EIA3 constructions;
- Section 12 gives the evaluation of ZUC on the resistance against cryptanalytic attacks;
- Section 13 concludes the evaluation;
- Annex A includes a list of external references that are related to the results in this report.

5 Background to the design and evaluation work

The security architecture for LTE is specified in ref. [1]. This includes various services that need to be provided by standardised cryptographic algorithms. In particular, two standardised algorithms are required for the radio interface, namely:

- EEA – Encryption algorithm
- EIA – Integrity algorithm

Before this work began, two encryption and integrity algorithm sets had already been developed and standardised for LTE. The first set, 128-EEA1 and 128-EIA1, is based on SNOW 3G; the second, 128-EEA2 and 128-EIA2, is based on AES. (The prefix "128-" indicates that the algorithms take a 128-bit secret key.)

3GPP SA3 agreed in May 2009 on a requirement for a third encryption and integrity algorithm set – one designed in China, so that the Chinese authorities would permit its use in that country.

The resulting algorithm set is based on a core stream cipher algorithm named ZUC, after Zu Chongzhi, the famous Chinese scientist from history. The algorithms were designed by experts at the Data Assurance and Communication Security Research Center (DACAS) of the Chinese Academy of Sciences.

Of course, "an algorithm from China" is not enough of a requirement. It was agreed that a robust, three-phase evaluation programme would be followed:

- evaluation by an ETSI SAGE task force;

- evaluation by two funded teams of academic experts, delivering their results ([32],[33]) to the ETSI SAGE task force;

- after that evaluation, if the task force recommended that the algorithm (modified or not) is suitable for acceptance into the standard, then a public evaluation phase would take place before final standardisation.

The SAGE task force set high security goals for the algorithm set. To recommend the algorithms, it was not sufficient merely that no practical attacks were found during the evaluation. Rather, the task force required that the algorithms should be judged to have a high security margin, and also that the design rationale behind all components of the algorithm should be clear and transparent.

6 Versions of these algorithms / evaluation history

Three versions of the ZUC, 128-EEA3 and 128-EIA3 algorithm have existed, which we will refer to by dates:

- Jan 2010 version: this was the version that was evaluated by the two funded teams, as described in section 5 [32], [33].

- June 2010 version: this was the version initially published on the GSMA web site (http://gsmworld.com/our-work/programmes-and-initiatives/fraud-and-security/gsm_security_algorithms.htm) for public evaluation, and subject to public evaluation particular from August-December 2010, including the ZUC Workshop in Beijing in December. The individual documents have version numbers as follows: 128-EEA3 & 128-EIA3 Specification, version 1.4; ZUC Specification, version 1.4; Implementors' Test Data, version 1.0. There were two small changes to ZUC from the Jan 2010 version, which are described in section 8.1.3; the 128-EEA3 and 128-EIA3 constructions were unchanged.

- Jan 2011 version: this is the version primarily described in this document, and referenced as [2] and [3]. A few small changes have been made to ZUC and to the 128-EIA3 construction – these are described in sections ??.

All reference to the algorithms in this document is to the Jan 2011 version unless otherwise stated.

7 Algorithm parameters

The text in this section is taken directly from [1].

7.1 EEA – Encryption algorithm

The input parameters to the ciphering algorithm are a 128-bit cipher key named KEY, a 32-bit COUNT, a 5-bit bearer identity BEARER, the 1-bit direction of the transmission i.e. DIRECTION, and the length of the keystream required i.e. LENGTH. The DIRECTION bit shall be 0 for uplink and 1 for downlink.

Figure 6.1 illustrates the use of the ciphering algorithm EEA to encrypt plaintext by applying a keystream using a bit per bit binary addition of the plaintext and the keystream. The plaintext may be recovered by generating the same keystream using the same input parameters and applying a bit per bit binary addition with the ciphertext.

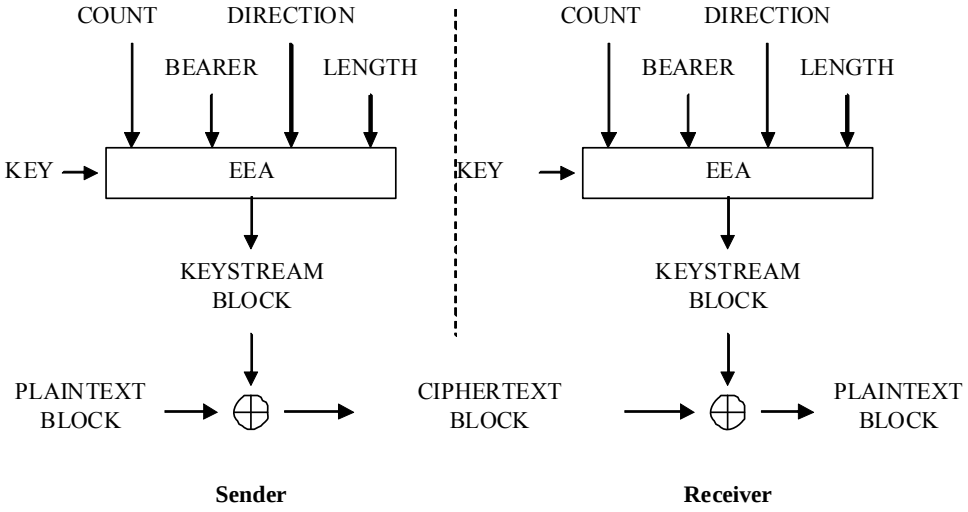

Figure 6.1: Ciphering of data

Based on the input parameters the algorithm generates the output keystream block KEYSTREAM which is used to encrypt the input plaintext block PLAINTEXT to produce the output ciphertext block CIPHERTEXT.

The input parameter LENGTH shall affect only the length of the KEYSTREAM BLOCK, not the actual bits in it.

7.2 EIA – Integrity algorithm

The input parameters to the integrity algorithm are a 128-bit integrity key named KEY, a 32-bit COUNT, a 5-bit bearer identity called BEARER, the 1-bit direction of the transmission i.e. DIRECTION, and the message itself i.e. MESSAGE. The DIRECTION bit shall be 0 for uplink and 1 for downlink. The bit length of the MESSAGE is LENGTH.

Figure 6.2 illustrates the use of the integrity algorithm EIA to authenticate the integrity of messages.

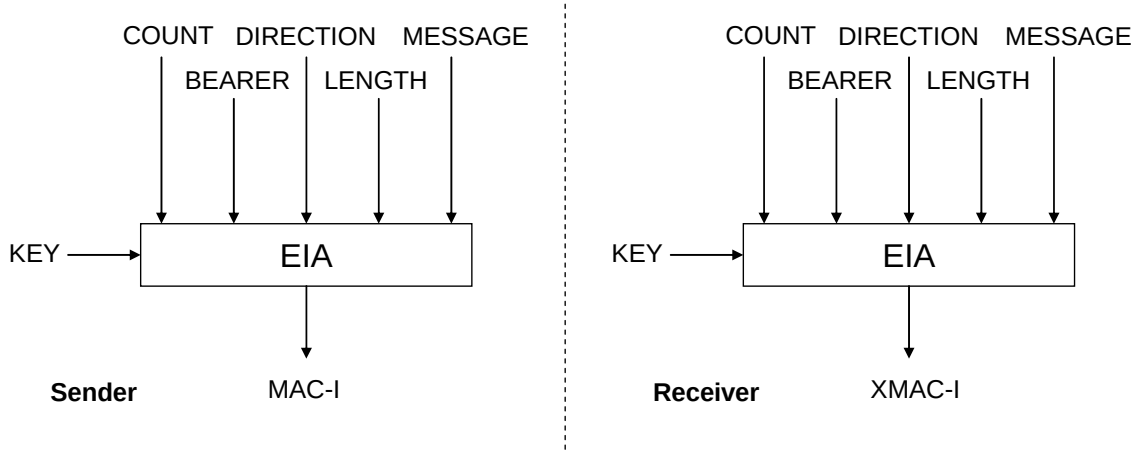

Figure 6.2: Derivation of MAC-I/NAS-MAC (or XMAC-I/XNAS-MAC)

Based on these input parameters the sender computes a 32-bit message authentication code (MAC-I/NAS-MAC) using the integrity algorithm EIA. The message authentication code is then appended to the message when sent. The receiver computes the expected message authentication code (XMAC-I/XNAS-MAC) on the message received in the same way as the sender computed its message authentication code on the message sent and verifies the data integrity of the message by comparing it to the received message authentication code, i.e. MAC-I/NAS-MAC.

8 Summary of the 128-EEA3 and 128-EIA3 algorithms

The detailed specifications of the 128-EEA3 and 128-EIA3 algorithms can be found in ref. [2] and ref.[3]. This section includes a brief overview of the 128-EEA3 and 128-EIA3 designs. The basic building block for both 128-EEA3 and 128-EIA3 is the stream cipher algorithm ZUC with an internal state of 560 bits initialized from a 128-bit cipher key and a 128-bit initialization vector.

8.1 The stream cipher ZUC

8.1.1 The structure of ZUC

The structure of ZUC is depicted in the following diagram:

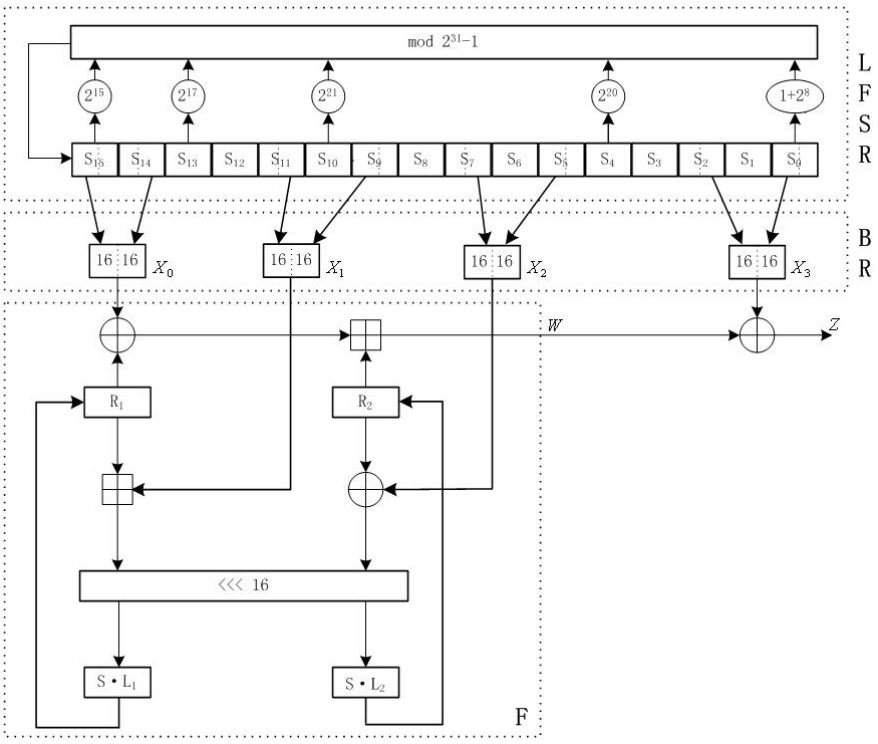

Figure 8.1: The structure of ZUC

ZUC consists of a Linear Feedback Shift Register (LFSR), a bit-reorganization and a nonlinear function F. The LFSR is constructed from 16 register cells, each holding 31 bits, and the feedback is defined by a primitive polynomial over the finite field GF(2^{31}-1) . The bit-reorganization extracts 128 bits from the cells of the LFSR and forms four 32-bit words which will be used by the nonlinear function F and the output of the keystream. The nonlinear function F is based upon two 32-bit memory cells R_1 and R_2. The nonlinear function F takes 3 of 32-bit words from the bit-reorganization as its inputs and uses two S-boxes S_0 and S_1. It also involves different operations such as the exclusive-OR, the cyclic shift and the addition modulo 2^{32}. See ref. [3] for details on the specification of ZUC.

8.1.2 Changes from the June 2010 version

At the 2010 ZUC workshop, analysing the June 2010 version of the algorithms, Bing Sun et al. [35] pointed out that the ZUC initialization process does not preserve key entropy. The same point was made by one of the funded evaluation teams in private discussion. At the rump session of Asiacrypt 2010, Hongjun Wu [36] pointed out a different cause for the key entropy loss of ZUC initialization, which led to a chosen IV attack against ZUC and 128-EEA3.

In the June 2010 version of ZUC, Z rather than W was included in the LFSR feedback (refer to Figure 8.1); this causes the entropy loss observed in [35]. Also, this combining was done by XOR rather than by addition mod 2^{31}-1; this causes the entropy loss observed in [36]. In the Jan 2011 versions these two problems are fixed: we use W rather than Z as input to the LFSR feedback, and we perform the combination mod 2^{31}-1. The use of W rather than Z in fact reverses a change that was made when moving from the Jan 2010 to the June 2010 version.

A proof is given in section 12.1 to show that the Jan 2011 modifications preserve the entropy of the initial keys during the ZUC initialization, which avoids the weakness of possible key collision.

8.1.3 Changes from the Jan 2010 version to the June 2010 version

Refer again to Figure 8.1. In the Jan 2010 version of ZUC, the arrows down from the LFSR to $X_0 \ldots X_3$ came from different LFSR half-words. Specifically, we had $X_0=s_{15H} \| s_{14L}$; $X_1=s_{12L} \| s_{8H}$; $X_2=s_{7H} \| s_{5L}$; and $X_3=s_{2L} \| s_{0H}$. This meant that there was a 16-bit overlap between the LFSR bits used in two successive time instances (s_{8H} at one time instance became s_{7H} at the next). Although we identified no way to exploit that property, nevertheless it seemed better to avoid it.

The other change from Jan 2010 to June 2010 was referred to already in section 8.1.2. In the Jan 2010 version, W rather than Z was combined into the LFSR feedback during ZUC initialisation. This change was made to improve resistance against a particular kind of attack that a student had been investigating. But in fact the "attack" turned out to have been based on a false assumption anyway, so there was no need to improve resistance to it. And as we saw in section 8.1.2, the change weakened the algorithm – so it has been reversed in the Jan 2011 version.

8.2 Confidentiality function 128-EEA3

The confidentiality algorithm (128-EEA3) encrypts and decrypts frames using ZUC as a synchronous stream cipher. Ref. [2] defines how the system parameters COUNT, BEARER and DIRECTION are used together with the confidentiality key (CK) to initialize the keystream generator.

The output from ZUC consists of 32-bit words that are XORed to the corresponding Input Bit Stream (IBS).

The main stream cipher principles of 128-EEA3 are shown in Figure 8.2. The produced cipher text block is denoted as Output Bit Stream (OBS).

For decryption the same scheme is used to recover the plain text block (IBS) from the received cipher text (OBS). Sender and receiver will synchronize for each frame using the frame counter COUNT.

Apart from the changes to ZUC itself, the Jan 2011 version of the 128-EEA3 algorithm is unchanged from the Jan 2010 and June 2010 versions.

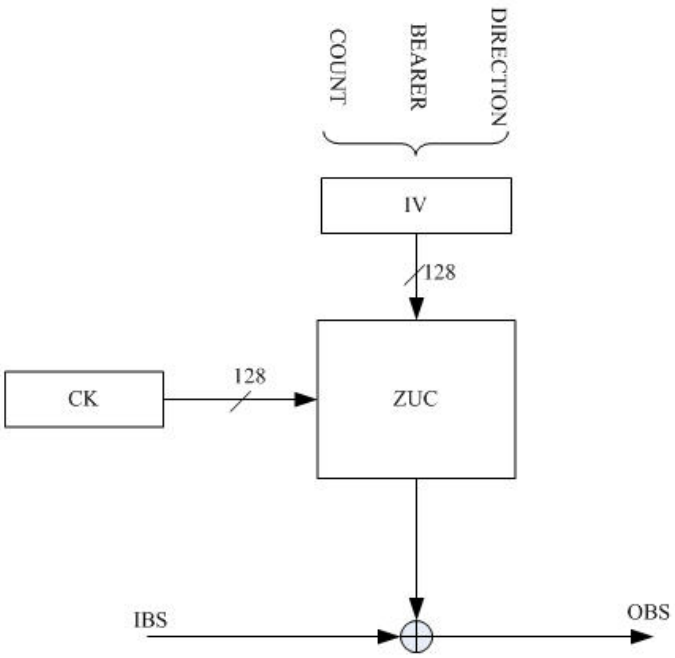

Figure 8.2: Principles of the 128-EEA3 encryption operation

8.3 Integrity function 128-EIA3

8.3.1 The 128-EIA3 construction

The integrity algorithm (128-EIA3) computes a 32-bit Message Authentication Code (MAC) on an input message using an integrity key IK. It is required that the message should be between 1 and 20000 bits in length (but the algorithm supports longer messages if needed). The 128-EIA3 algorithm is based on universal hashing and one-time-pad masking.

Prior to processing the message, the ZUC generator is initialized with the integrity key IK and the initialization vector IV, and a keystream with length 2 words more than that of the message is generated.

The message is padded with a bit 1. Set the initial value of an accumulator variable T to be 0. For each bit of the padded message, if the *i*-th bit of message is 1, then the accumulator accumulates T by XOR with the word defined by the successive 32 bits starting from the *i*-th bit of the keystream. Finally T is masked by a whole output word of ZUC that has not been used at all in computing T as the MAC of the message.

The structure of 128-EIA3 is depicted in Figure 8.3.

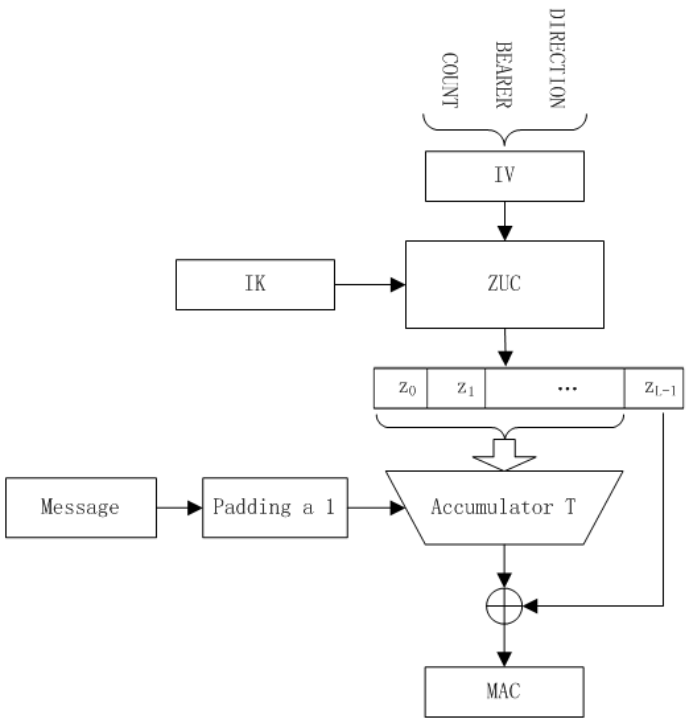

Figure 8.3: Principles of the 128-EIA3 MAC computation

8.3.2 Changes from the June 2010 version

The 128-EIA3 construction used in the Jan 2010 and June 2010 versions used a bad choice of mask, which did not quite fit the well known proof schema [8], [9] for MAC functions based on universal hash functions. This problem was identified by Thomas Fuhr et al [37], who demonstrated a forgery attack. In the Jan 2011 version we choose a mask consisting of a whole ZUC output word, instead of (as before) a 32-bit string potentially spanning two ZUC output words. The universal hash function security proof now applies correctly.

9 Overall design rationale for ZUC

The essential design goals for the third suite of 3GPP confidentiality and integrity algorithms are that the algorithms should:

- provide a high level of security within the 3GPP context;
- meet the implementation requirements deemed by 3GPP — in particular, allow a low power, low gate count implementation in hardware;
- the fundamental cryptographic principles be substantially different from the AES-based and SNOW 3G-based constructions – so that progress in cryptanalysis towards one of the algorithms should not directly lead to attacks against the other.

The ZUC algorithm adopts a linear feedback shift register (LFSR) that generates *m*-sequences over the prime field $GF(2^{31}-1)$ as source sequences of the algorithm, which obviously have substantial differences from traditional stream ciphers that are based on *m*-sequences over the finite field GF(2) or its extension field $GF(2^n)$. The *m*-sequences over the prime field $GF(2^{31}-1)$ are not highly structured over GF(2) – for example, they have high linear complexity over

GF(2), low advantage of bit coincidence between mod 2^{31}-1 and mod 2 sums, and many other good cryptographic properties. This component of *m*-sequence generator has significant contribution to the ZUC algorithm against bit-oriented cryptographic attacks, including fast correlation attacks, linear distinguishing attacks, and algebraic attacks.

The design of the nonlinear function *F* adopts some structures in block ciphers, namely S-boxes and a mixture of XOR and modulo addition. This design gives protection against many well known cryptographic attacks, including guess and determine attacks, fast correlation attacks and linear distinguishing attacks. Between the nonlinear function *F* and the feedback shift register, there is a bit-reorganization layer. Since the algebraic structures of the linear feedback shift register, the bit-reorganization layer, and the nonlinear function *F*, are entirely different, the combination of them is expected to enhance the overall security. The LFSR on its own can of course be analysed over GF(2^{31}-1), but the additional layers make it hard to pursue that analysis throughout the cipher as a whole.

The following types of attacks against the stream cipher ZUC have been particularly considered:

- algebraic attacks;
- guess and determine attacks;
- fast correlation attacks;
- distinguishing attacks;
- attacks on the key loading and initialization elements.

Attacks against the 128-EEA3 and 128-EIA3 constructions are also considered.

10 Design and evaluation of ZUC components

10.1 Design of the LFSR

As explained in section 8.3.2, ZUC uses an LFSR operating over GF(2^{31}-1) instead of one operating over GF(2) or GF(2^n). In this section we analyse some properties of LFSRs over GF(2^{31}-1), and explain how the characteristic polynomial of the LFSR was selected.

10.1.1 The binary coordinates are equivalent by shift

Let $p=2^{31}$-1 and GF(p) be the finite field with p elements. For any given primitive polynomial $f(x)$ over GF(p), let $\underline{a}$ be an *m*-sequence over GF(p) generated by $f(x)$. Then it can be written in 2-adic representation, i.e., in terms of its binary coordinate sequences as:

$$\underline{a}=\underline{a}_0+\underline{a}_1 2+\ldots+\underline{a}_{30}2^{30}, \quad (1)$$

where $\underline{a}_i$ ($0 \le i \le 30$) is a binary sequence composed of 0's and 1's, which is called the i-coordinate sequence of $\underline{a}$.

Denote by $G(f(x),p)$ the set of all sequences generated by $f(x)$ over GF(p). For any $\underline{a} \in G(f(x),p)$, we have

$$2\underline{a}=\underline{a}_{30}+\underline{a}_0 2+\ldots+\underline{a}_{29}2^{30} \bmod p \quad (2)$$

On the other hand, for any $\underline{a}=(a_0,a_1,a_2,\ldots) \in G(f(x),p)$, it can be represented as follows:

$$\underline{a}=(\mathrm{Tr}(\lambda\alpha^0),\mathrm{Tr}(\lambda\alpha^1),\mathrm{Tr}(\lambda\alpha^2), \ldots)$$

where α is a root of $f(x)$, $\lambda \in \mathrm{GF}(p)(\alpha)$, and Tr() is a trace function from $\mathrm{GF}(p)(\alpha)$ to $\mathrm{GF}(p)$. Hence we get

$$\begin{aligned} 2\,\underline{a} &= (2\mathrm{Tr}(\lambda\alpha^0), 2\mathrm{Tr}(\lambda\alpha^1), 2\mathrm{Tr}(\lambda\alpha^2), \ldots) \\ &= (\mathrm{Tr}(2\lambda\alpha^0), \mathrm{Tr}(2\lambda\alpha^1), \mathrm{Tr}(2\lambda\alpha^2), \ldots) \\ &= (\mathrm{Tr}(\lambda\alpha^m), \mathrm{Tr}(\lambda\alpha^{m+1}), \mathrm{Tr}(\lambda\alpha^{m+2}), \ldots) \\ &= (a_m, a_{m+1}, a_{m+2}, \ldots) \end{aligned} \quad (3)$$

where $m=5/31\,(p^{\deg(f(x))}-1)$ such that $\alpha^m=2$.

This means that the 1-coordinate sequence of $\underline{a}$ is obtained by shifting its 0-coordinate sequence by m bits to the left, and the 2-coordinate sequence of $\underline{a}$ is obtained by shifting its 1-coordinate sequence by m bits to the left, and so on, and the 0-coordinate sequence of $\underline{a}$ is obtained by shifting its 30-coordinate sequence by m bits to the left.

Moreover, since the coordinate sequences are all periodic, they are equivalent by shift.

10.1.2 The LFSR sequences are entropy-lossless under the modulo 2 operation

In this section we will prove that, the set $G(f(x),p)$ of the m-sequences generated by $f(x)$, are entropy-lossless under the modulo 2 operation. More precisely, we have

Theorem 1 [34] *Let $f(x)$ be a primitive polynomial over* GF*(p), and let $G(f(x),p)$ denote the set of sequences over* GF*(p) generated by $f(x)$. Then for any $\underline{a},\underline{b} \in G(f(x),p)$, we have that, $\underline{a}=\underline{b}$ if and only if $\underline{a}$* mod 2=$\underline{b}$ mod 2 *holds.*

By theorem 1, and taking into account the properties in section 10.1.1 that all the coordinate sequences are equivalent by shift, we know that, the binary sequences resulted from the modulo 2 operation are entropy-lossless.

10.1.3 On the period and linear complexity of the coordinate sequences

From section 10.1.2 we know that, the binary sequences derived from the modulo 2 operation of the sequences over GF(p) generated by a primitive polynomial $f(x)$ are entropy-lossless. This means that every coordinate sequence is also entropy-lossless, which also means that it has the same period with the original sequences, i.e.

$$\mathrm{T}(\underline{a}_i)=\mathrm{T}(\underline{a})=p^{\deg(f(x))}-1,\ i=0,1,\ldots,30, \quad (4)$$

where T() denotes the period of sequences.

Therefore a chosen primitive polynomial $f(x)$ with suitable degree can guarantee that the period of each coordinate sequence is sufficiently large. For example, if $\deg(f(x))=16$, then the period of each coordinate sequence of sequences generated by $f(x)$ is $p^{16}-1 \approx 2^{496}$.

From section 10.1.1 we know that the coordinate sequences are equivalent by shift, which means that these sequences have the same linear complexity. It is pointed out in ref [4] that the linear complexity of the coordinate sequences is

$$LC(\underline{a}_i)=\tau\,(p^{16}-1)/(p-1),\ \ i=0,1,\ldots,30, \tag{5}$$

where τ is the linear complexity over GF(2) of the binary sequence $\{\zeta^t \bmod 2\}_{t\geq 0}$, where ζ is a primitive element of the finite field GF(p). In most cases, τ is about a half of p. Hence the linear complexity of the coordinate sequences $LC(\underline{a}_i)$ is about a half of their period.

10.1.4 On the advantage of bit coincidence between additions mod *p* and mod 2

Let $J=\{1,2,\ldots,p\}$, and $x_1,x_2,\ldots,x_k$ be k independent probabilistic variables over J complying with uniform probability distribution. Let their addition modulo p be as follows:

$$y= x_1+x_2+\ldots+x_k \bmod p,$$

where $y\in J$. Below we will discuss the bit coincidence between two probabilistic variables.

Let $u= x_1\oplus x_2\oplus \ldots\oplus x_k$. Write both y and u in their 2-adic representation as

$$y=y_0+y_1 2+y_2 2^2+\ldots+y_{30}2^{30},$$

$$u=u_0+u_1 2+u_2 2^2+\ldots+u_{30}2^{30},$$

where y_i and u_i will take value 0 or 1, $i=0,1,\ldots,30$. Then the advantage of the bit coincidence between y_i and u_i are defined as

$$\Pr(y_i=u_i)-1/2.$$

It is also called the i-th bit coincidence between y and u, because y_i and u_i are the i-th coordinate of y and u respectively. Now we have the following conclusion:

Theorem 2 *For any given integer k, let ϵ_i be the i-th bit coincidence between y and u, $i=0,1,\ldots,30$, i.e.,*

$$\epsilon_i=\Pr(y_i=u_i)-1/2.$$

Then we have

$$\epsilon_0=\epsilon_1=\ldots=\epsilon_{30}=\epsilon(k,p), \tag{6}$$

where $\epsilon(k,p)$ is a constant that is only related to k and p.

Proof: We know that $2a\equiv a<<<_{31}1 \bmod p$ for arbitrary $a\in J$, where $<<<_{31}$ denotes the cyclic shift of 31-bit words to the left. So the i-th bit of $\sum(x_j)$ is the same as the $(i+1)$-th bit of $\sum(2x_j)$, and also the i-th bit of $\bigoplus(x_j)$ is the same as the $(i+1)$-th bit of $\bigoplus(2x_j)$. Hence $\epsilon_i = \epsilon_{i+1}$ holds for all i. ■

Let

$$J_{k,t}=\{(x_1,x_2,\ldots,x_k)|\ x_1+x_2+\ldots+x_k \leq t, 1\leq x_i\leq p, 1\leq i\leq k\}$$

and denote by $N_{k,t}$ the number of elements in the set $J_{k,t}$. Then we have:

1. When k=1:

$$N_{1,t}=\begin{cases} 0 & \text{if } t<1, \\ t & \text{if } 1\leq t\leq p, \\ p & \text{if } t>p. \end{cases} \tag{7}$$

2. When k>1:

$$N_{k,t}=\sum_{1\leq i\leq p} N_{k-1,t-i}. \tag{8}$$

By equations (7) and (8), it is possible to compute the value of any $N_{k,t}$ in theory. Now we give a specific method to compute $\epsilon(k,p)$ using $N_{k,t}$.

Theorem 3 *For any integer k,*$1\leq k<p$*, let* $\epsilon(k,p)$ *and* $N_{k,t}$ *be as defined above. Then we have*

$$\epsilon(k,p)=1/p^k \sum_{0\leq i\leq \lfloor k/2\rfloor}(N_{k,(2i+1)p-1}- N_{k,2ip-1})-1/2 \tag{9}$$

Proof: For any $x_i\in J$, $1\leq i\leq k$, write

$$\sum_{1\leq i\leq k} x_i = X_0+X_1p$$

for integers X_0, X_1, where $0\leq X_0< p$. Then $u_0= X_0+X_1 \bmod 2$ and $y_0= X_0 \bmod 2$. So they are equal if and only if $X_1 \bmod 2=0$, that is, if and only if

$$2ip\leq \sum_{1\leq j\leq k} x_j <(2i+1)p$$

for some integer $i\geq 0$, and the number of x_i (i=1,2,…,k) holding the above inequalities is

$$N_{k,(2i+1)p-1}- N_{k,2ip-1}$$

However it is straightforward to show by induction that $N_{k,t} = p^k$ for $t\geq kp$, so we only need to consider $i\geq 0$ where $2ip-1< kp$ holds, that is, $2ip\leq kp$, or $i\leq \lfloor k/2\rfloor$. Hence the required result holds for $\epsilon(k,p)$. ■

By using equation (9), we can compute the advantage of bit coincidence for the cases when k=2,3,4,5,6,7 and 8, as shown in Table 10.1 (The values in Table 10.1 have been corrected by the evaluation report [33]).

Table 10.1: The values of $\epsilon(k,p)$, where k=2,3,4,5,6,7,8

k	2	3	4	5	6	7	8
$\epsilon(k,p)$	-2^{-32}	$-2^{-2.585}$	$-2^{-33.685}$	$2^{-3.907}$	$2^{-33.322}$	$-2^{-5.212}$	$-2^{-33.890}$

From Table 10.1 it is seen that, when k is even, then the bit coincidence is small, and when k is odd, then the bit coincidence can be significantly large, which is determined by the value of k, and has little to do with the value of p.

10.1.5 Fast implementations of the addition and multiplication over GF(p)

Since the modulus $p=2^{31}-1$ is a special prime, the addition and multiplication over GF(p) has a fast implementation shown as below.

Let $x, y \in$ GF(p). Write $x+y=z+c2^{31}$ for some integer $c \in \{0,1\}$. Then

$$x+y \bmod p = z+c .$$

This means that the modulo p addition can avoid the modulo p operation. (We are using the ZUC convention that mod p integers are in the range 1 to p, rather than 0 to p-1.)

For arbitrary integer $0 \leq k \leq 30$, we have

$$2^k x \bmod p = x <<<_{31} k$$

So when the Hamming weight of the operand a is small, for example, $a=2^i+2^j+2^k$, the multiplication ax over GF(p) can be calculated with the following method:

$$ax \equiv (x <<<_{31} i)+(x <<<_{31} j)+ (x <<<_{31} k) \bmod p.$$

This means that the modulo p multiplication can be done by cyclic shifts and modulo p addition, hence the modulo p operation does not need to be actually implemented.

10.1.6 Choice of the primitive polynomial $f(x)$

In sections 10.1.1-10.1.5, some properties of m-sequences over GF(p) are introduced, and it shows that we should choose some primitive polynomial $f(x)$ with even number of terms whose nonzero coefficients have as low Hamming weights as possible. More precisely, the choice of the primitive polynomial $f(x)$ should meet the following criteria:

1. The Hamming weight of each nonzero coefficient is as low as possible;
2. The sum of the Hamming weights of nonzero coefficients (ignoring the x^{16} term) is an even number;
3. The coefficient of the term with the second highest degree should be nonzero;
4. The difference of degrees of term with nonzero coefficients (ignoring the x^{16} term) should be pair-wise different;
5. The difference of positions of 1's of nonzero coefficients in their 2-adic representation should be pair-wise different.

According to the above criteria, the following polynomial is chosen

$$f(x)=x^{16}-(2^{15}x^{15}+2^{17}x^{13}+2^{21}x^{10}+2^{20}x^4+(2^8+1)). \quad (10)$$

Since $f(x)$ is a primitive polynomial over GF(p), the LFSR will generate an m-sequence with period $p^{16}-1 \approx 2^{496}$. By section 10.1.3, the GF(2) linear complexity of the coordinate sequences of those generated by $f(x)$ is about a half of their period, i.e.,2^{495}.

10.2 Design of the bit-reorganization

The bit-reorganization layer is the connection between the LFSR and the nonlinear function F, and it extracts 128 bits from the cells of the LFSR and forms four 32-bit words. The design of the bit-reorganization mainly refers to the following criteria:

1. Suitable for software implementation;

2. The four 32-bit words from the bit-reorganization have good randomness in the statistical sense;

3. The number of the overlapping bits of four 32-bit words in successive times is small.

Based on the above criteria, the bit-reorganization works as follows:

a) $X_0 = s_{15H} \| s_{14L}$;

b) $X_1 = s_{11L} \| s_{9H}$;

c) $X_2 = s_{7L} \| s_{5H}$;

d) $X_3 = s_{2L} \| s_{0H}$,

where X_i (i=0,1,2,3) are the output of the bit-reorganization, and the subscripts H and L of some cell s indicate the high 16 bits and low 16 bits of s respectively.

As for the bit-reorganization constructed above, within two consecutive times, i.e., time t and time t+1, there is one bit in common among the constructed words; in time t and t+2, there are 4 bits in common among the constructed words; in time t and t+3, there are 17 bits in common among the constructed words; in time t and t+4, there are 33 bits in common among the constructed words. Common bits in other time intervals can be calculated easily.

Note: in the original ZUC design, the 128 bits were extracted from different cells of the shift register (i.e. in Figure 8.1, the eight arrows down from the LFSR to the BR block came from different "half words" in the register cells). It was noted that, at times t and t+1, there were 17 bits in common among the constructed words. Although no attack was identified to exploit this property, it was nevertheless undesirable; hence the arrow positions have been changed, reducing the overlap at times t and t+1 to a single bit.

In ZUC algorithm, since elements in GF(p) are defined to be within the set $\{1,2,\ldots,p\}$, therefore during the feedback process of the 16-stage LFSR, the value 0 should be replaced by p. It is noted that the LFSR will generate an m-sequence of period $p^{16}-1$. So in one period of such sequence, there will be no such state when all the cell values are equal to p. It is easy to check the following conclusion:

Theorem 4 $\Pr(s_i=p)=(p^{15}-1)/(p^{16}-1)$*, and for any* $1 \leq a \leq p-1$ *we have*

$$\Pr(s_i=a)=p^{15}/(p^{16}-1). \quad (11)$$

From Theorem 4 we can get the following conclusion immediately:

Theorem 5

$$\Pr(s_{i\mathrm{H}}=0)=\Pr(s_{i\mathrm{L}}=0)=(2^{15}-1)p^{15}/(p^{16}-1),$$

$$\Pr(s_{i\mathrm{H}}=2^{16}-1)=\Pr(s_{i\mathrm{L}}=2^{16}-1)=(2^{15}p^{15}-1)/(p^{16}-1),$$

$$\Pr(s_{i\mathrm{H}}=a)=\Pr(s_{i\mathrm{L}}=a)=2^{15}p^{15}/(p^{16}-1),$$

where a is a 16-bit binary string which is neither all-zero nor all-one.

Let X_0, X_1, X_2, X_3 be the four 32-bit words obtained by the bit-reorganization process.

By theorem 5 and the well-known statistical properties of *m*-sequences, we have the following conclusions:

Corollary 1 *Let a and b be two arbitrary 16-bit binary strings which are neither all-zero nor all-one. Then*

$$\Pr(X_0=(a,b))= 2^{15}p^{15}/(p^{16}-1)\times 2^{15}p^{15}/(p^{16}-1) \tag{12}$$

Corollary 2 *Let a be an arbitrary 16-bit binary string which is neither all-zero nor all-one,* **1** *is a 16-bit all-one binary string. Then we have*

$$\Pr(X_0=(a,\mathbf{1}))= \Pr(X_0=(\mathbf{1}, a))= 2^{15}p^{15}/(p^{16}-1)\times(2^{15}p^{15}-1)/(p^{16}-1) \tag{13}$$

Corollary 3 *Let a be an arbitrary 16-bit binary string which is neither all-zero nor all-one,* **0** *is a 16-bit all-zero binary string. Then we have*

$$\Pr(X_0=(a,\mathbf{0}))= \Pr(X_0=(\mathbf{0}, a))= 2^{15}p^{15}/(p^{16}-1)\times(2^{15}-1)p^{15}/(p^{16}-1) \tag{14}$$

Corollary 4 *Let* **1** *be a 16-bit all-one string,* **0** *be a 16-bit all-zero string, then we have*

$$\Pr(X_0=(\mathbf{0},\mathbf{1}))= \Pr(X_0=(\mathbf{1},\mathbf{0}))= (2^{15}p^{15}-1)/(p^{16}-1)\times(2^{15}-1)p^{15}/(p^{16}-1) \tag{15}$$

Corollary 5 *Let* **0** *be a 32-bit all-zero string, then we have*

$$\Pr(X_0=\mathbf{0})= (2^{15}-1)p^{15}/(p^{16}-1)\times(2^{15}-1)p^{15}/(p^{16}-1) \tag{16}$$

Corollary 6 *Let* **1** *be a 32-bit all-one string, then we have*

$$\Pr(X_0=\mathbf{1})= (2^{15}p^{15}-1)/(p^{16}-1)\times(2^{15}p^{15}-1)/(p^{16}-1) \tag{18}$$

Corollary 7 *The random variables X_0, X_1, X_2* and *X_3 have the same probability distribution.*

Based on the above corollaries, we can roughly treat X_0, X_1, X_2, X_3 as having uniform probability distribution. With respect to their distinguishability with uniform probability distribution, it can be reflected from the variance. Take X_0 as an example, we have

$$\varepsilon(X_0)=\sum_{a=0}^{2^{32}-1}[P(X_0=a)-\frac{1}{2^{32}}]^2 \approx 2^{-77} \tag{19}$$

The above means that it needs at least $N=\varepsilon^{-1}\approx 2^{77}$ instances of the variable X_0 before it can be distinguished from a real random one.

10.3 Design of the nonlinear function *F*

The nonlinear function *F* is a compression function from 96 bits to 32 bits. Its design adopts some structures from block cipher design. Considering both security and performance requirements, the design of the nonlinear function *F* mainly refers to the following criteria:

1. Take 96 bits as input and output a 32-bit word;

2. The nonlinear function *F* should carry memories;

3. The nonlinear function *F* should use S-boxes in order to possess high nonlinearity and other cryptographic properties;

4. The nonlinear function *F* should use linear transforms with good diffusion;

5. The output sequences of the nonlinear function *F* should be balanced and have high unpredictability;

6. The nonlinear function *F* should be suitable for software and hardware implementations;

7. The cost of hardware implementations of the nonlinear function *F* should be low.

10.3.1 Design of the S-boxes S_0 and S_1

Two S-boxes are used in the nonlinear function *F* and named S_0 and S_1 respectively. Since the LFSR is shown to have good behaviour in resistance against algebraic attacks over GF(2), thus the algebraic immunity was not the highest priority in the design of the S-boxes.

10.3.1.1 Design of S-box S_0

The design of the S-box S_0 mainly refers to the following three criteria:

1. The cost of its hardware implementation is low;

2. S_0 should have high nonlinearity;

3. S_0 should have low differential uniformity.

Based on the above consideration, the S-box S_0 is designed using a Feistel structure, see Figure 10.1.

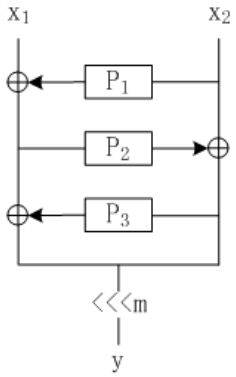

Figure 10.1: The structure of the S-box S_0

In Figure 10.1, both x_1 and x_2 are 4-bit strings, m=5, and P_1,P_2,P_3 are transforms over GF(16), which are defined as in Tables 9.2, 9.3 and 9.4 respectively.

Table 10.2: The transform P_1

Input	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Output	9	15	0	14	15	15	2	10	0	4	0	12	7	5	3	9

Table 10.3: The transform P_2

Input	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Output	8	13	6	5	7	0	12	4	11	1	14	10	15	3	9	2

Table 10.4: The transform P_3

Input	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Output	2	6	10	6	0	13	10	15	3	3	13	5	0	9	12	13

As for the S-box S_0, it is easy to verify that its nonlinearity, differential uniformity, algebraic degree and algebraic immunity are 96, 8, 5 and 2 respectively. Due to its algebraic structure, no more than 11 linearly independent quadratic equations in eight input and eight output bits can be established.

10.3.1.2 Design of S-box S_1

The design of the S-box S_1 mainly refers to the following two criteria:

1. The nonlinearity of the S-box S_1 is as high as possible;

2. The differential uniformity of the S-box S_1 is as low as possible.

According to the above criteria, the S-box S_1 is based on the inversion over the finite field GF(256) defined by the binary polynomial $x^8+x^7+x^3+x+1$, and composes one affine function after the inversion. More precisely, the S-box S_1 can be written as follows

$$S_1=Mx^{-1}+B \tag{20}$$

where B=0x55 and M is a matrix of size 8×8 and defined as below:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}.$$

Since the S-box S_1 is affine equivalent to that of the advanced encryption standard AES, thus S_1 has many properties same as that of AES, including nonlinearity, differential uniformity, algebraic degree and algebraic immunity. More precisely, its nonlinearity, differential uniformity, algebraic degree and algebraic immunity are 112, 4, 7 and 2 respectively. Due to its algebraic structure, no more than 39 linearly independent quadratic equations in eight input and eight output bits can be established.

10.3.2 Design of the linear transforms L_1 and L_2

The goal of the linear transforms is mainly to provide good diffusion. Their designs mainly refer to the following two criteria:

1. L_1 and L_2 must have maximal differential and linear branch number.

2. L_1 and L_2 are suitable for software and hardware implementations.

As for the second criterion, one realization is to choose some functions composed of the operations exclusive-OR $\oplus$ and the cyclic shift <<<, which are friendly to software and hardware implementations. Let GF(2)[x] be the polynomial ring over the binary field GF(2). Consider the quotient ring GF(2)[x]/(x^{32}+1). A 32-bit string $a=a_{31}a_{30}\ldots a_0$ is identified with an element $a(x)$ according to the bijection φ from GF(2)^{32} to GF(2)[x]/(x^{32}+1):

$$a = a_{31}a_{30}\cdots a_0 \quad \rightarrow \quad a(x) = \sum_{i=0}^{31} a_i x^i.$$

It is easy to check that the following holds:

$$\varphi(a \oplus b) = \varphi(a) + \varphi(b),$$

$$\varphi(a <<< k) = \varphi(a)x^k.$$

So the linear transforms composed of the exclusive-OR and the cyclic shift can be viewed as polynomials over GF(2)[x]/(x^{32}+1), for example, the linear transform L_1 used in ZUC can be written in the polynomial form as:

$$L_1(x) = x^{24}+x^{18}+x^{10}+x^2+1.$$

Note that $x^{32}+1=(x+1)^{32}$, hence any polynomial $L(x)$ in GF(2)[x]/(x^{32}+1) is invertible if and only if $L(1) \neq 0$, and if and only if $w(L)$ is odd, where $w(L)$ is defined as the number of monomials in $L(x)$ and is called the weight of $L(x)$.

For any given polynomial $L(x)$ in GF(2)[x]/(x^{32}+1) with odd weight, we have $L^{32}(x)=L(x^{32})=L(1)=1$. So

$$L^{-1}(x)=L^{31}(x)=L(x^{16})L(x^8)L(x^4)L(x^2)L(x).$$

As for the first criterion, when a byte is viewed as a basic data unit, it is known that both maximum differential branch number and maximum linear branch number are 5. It is known that the weight of the linear transform $L(x)$ with maximum differential branch number and maximum linear branch number must be greater than or equal to 4, i.e., $w(L) \geq 4$. Since a linear transform $L(x)$ with $w(L)=4$ is not a permutation, therefore the weight of the linear transform $L(x)$ with maximum differential branch number and maximum linear branch number is at least 5.

By testing all the possible polynomials $L(x)$ of weight 5, only two polynomials $L_1(x)$ and $L_2(x)$ are found to meet the above conditions:

$$L_1(x) = x^{24}+x^{18}+x^{10}+x^2+1,$$

$$L_2(x)=x^{30}+x^{22}+x^{14}+x^8+1,$$

which are the ones used in ZUC. The inversions of the above polynomials $L_1(x)$ and $L_2(x)$ are as below:

$$L_1^{-1}(x)=x^{30}+x^{24}+x^{22}+x^{18}+x^{16}+x^{14}+x^{12}+x^8+x^4+x^2+1,$$

$$L_2^{-1}(x)=x^{30}+x^{28}+x^{24}+x^{20}+x^{18}+x^{16}+x^{14}+x^{10}+x^8+x^2+1.$$

The linear transforms $L_1(x)$ and $L_2(x)$ defined above both have maximum linear branch number and maximum differential branch number, both are equal to 5. It is found that the matrix representation of $L_1(x)$ and that of $L_2(x)$ over GF(2) happen to be transpose matrices of each other.

Note: The polynomial $L_1(x)$ is identical to a linear function in the block cipher SMS4.

11 128-EEA3 and 128-EIA3 constructions

11.1 128-EEA3 construction

The 128-EEA3 construction is identical to that of 128-EEA1 except that the underlying algorithms are different. The 128-EEA1 is based on the SNOW 3G algorithm, while 128-EEA3 is based on the ZUC algorithm. It is believed that the ZUC algorithm has different security resistance against known attacks, and hopefully it also has different resistance against unknown attacks.

11.2 128-EIA3 construction

Two possible principles for the design of 128-EIA3 were considered. One option was to re-use the SNOW 3G-based scheme for f9 developed for the first suite of 3GPP algorithms. However, the implementation of 128-EIA1 itself would take considerable amount of hardware resources except for SNOW 3G.

An alternative was to design an entirely different integrity algorithm. The 128-EIA3 algorithm just does so, and was designed to have a more efficient implementation. When ZUC is implemented for 128-EEA3, the 128-EIA3 construction takes very little extra hardware resources for implementation. The following provides its rationale behind 128-EIA3 construction.

There are three basic approaches to construct Message Authentication Codes (MAC): block-cipher-based construction, e.g. OMAC [5], hash-function-based construction e.g. HMAC [6], universal-hash-function-based construction, e.g. GMAC[7]. The construction of 128-EIA3 uses the third approach of MAC construction.

The use of universal hash functions for building a secure MAC has received much attention within the cryptographic community due to its nice security properties and efficient implementation. Typically, the MAC value is generated by one-time-pad masking of the univeral hashing value. We call this kind of MAC a CW-MAC [8]. Theoretic researches show that [8][9], if a universal hash is an ϵ-AXU, and the same nonce value (used in the generation of the mask) can never be re-used with the same secret key, then the probability of a successful attack against CW-MAC (i.e. forging a valid MAC of some new message) is no

greater than ϵ. Therefore, as long as ϵ is small enough and the one-time pad masking is secure, then CW-MAC is secure.

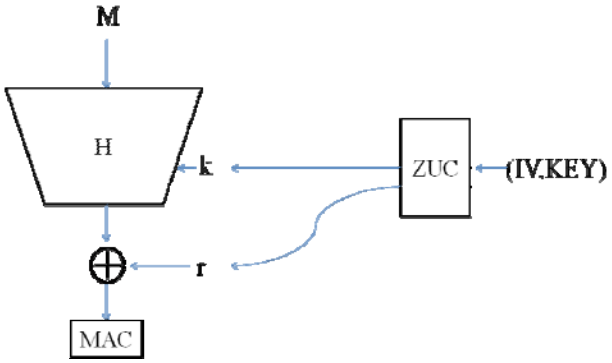

Figure 11.1: Structure of 128-EIA3

As shown in Figure 11.1, the core of 128-EIA3 is the universal hash function H and the keystream is generated by ZUC. We must stress that 128-EIA3 is a little different from CW-MAC. In CW-MAC, the one-time-pad mask is generated independently from the message, but in 128-EIA3, the mask is related to the length of the message. That is why the security proof of 128-EIA3 requires not only that H is AXU, but also that H is uniform.

In the Jan 2010 and June 2010 versions of 128-EIA3, the one-time pad key was the next 32 bits after the keystream used by H. Thomas Fuhr et al. [37] pointed out that this choice led to a flaw in the intended security proof, and made the construction vulnerable to an existential forgery attack with success probability ½. The identified weakness relates neither to H, nor to the underlying stream cipher ZUC, but only to the way the one-time-pad mask is derived from the keystream.

In the new version of 128-EIA3, the one-time-pad mask is a single output word of ZUC that has not been used at all in computing H. Although this is only a slight modification, this new version can resist the attack in [37], and can correctly be proved secure.

We make an assumption that the keystream generated by ZUC is indistinguishable from uniformly random.

Definition 1 *Let* $H:K\times D\rightarrow R$ *be a function, where K,D and R are the key space, the message space and the digest space respectively. Then H is called* ϵ*-AXU (*ϵ*-Almost-Xor-Universal) if for any* $x, x'\in D$, $x\neq x'$ *and* $y\in R$, *we have* $\Pr(H(k,x)\oplus H(k,x')=y)\leq \epsilon$, *where k is randomly chosen from K.*

This document uses the following universal hash function H: $K\times\{0,1\}^{*}\rightarrow\{0,1\}^{32}$ as the message authentication code function:

$$H(k,x)=\begin{pmatrix} k_1 & k_2 & \cdots & k_{m+1} \\ k_2 & k_3 & \cdots & k_{m+2} \\ \vdots & \vdots & \ddots & \vdots \\ k_{32} & k_{33} & \cdots & k_{m+32} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \\ 1 \end{pmatrix}, \tag{21}$$

where m is the bit length of the message, and it needs (m+32) keystream bits in order to handle an m-bit message.

The universal hash function above for 128-EIA3 is similar to the one based on Toeplitz matrix discussed in [10] and [11], with the exception that this new function can handle variable-input-length messages. Then the following conclusion holds.

Theorem 6 *Let $H(k, x)$ be defined as above. Then a) $H(k, x)$ is 2^{-32}-AXU. b) $H(k,x)$ is uniform in the sense that* $\Pr(H(k,x)=y)=2^{-32}$ *for any $x\in D$ and $y\in R$, where $k=k_1k_2\ldots k_{m+32}$ is a perfectly random keystream.*

Proof: a) Let $x=x_1x_2\ldots x_m$ and the 32×(m+32) matrix

$$A_x=\begin{pmatrix} x_1 & \cdots & \cdots & x_m & 1 & 0 & \cdots & 0 \\ 0 & x_1 & \cdots & \cdots & x_m & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & x_1 & \cdots & \cdots & x_m & 1 \end{pmatrix}.$$

Then

$$H(k,x)=A_x\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_{m+32} \end{pmatrix}.$$

For any x, x', $x\neq x'$ and $y\in \{0,1\}^{32}$, consider the following two cases:

1) If $|x|=|x'|$. Let $|x|=m$. Then

$$H(k,x)\oplus H(k,x')=(A_x+A_{x'})\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_{m+32} \end{pmatrix}.$$

Since $A_x+A_{x'}$ is a full rank matrix, and the dimension of the solution space is m, therefore

$\Pr(H(k,x)\oplus H(k,x')=y)\leq 2^m/2^{m+32}=2^{-32}$.

2) If $|x|\neq|x'|$. Let $m=|x|>|x'|=m'$, then

$$H(k,x')=(A_{x'},O)\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_{m+32} \end{pmatrix},$$

where O is a 32×(m-m') all-zero matrix. Similar to the case 1), it is known that $A_x+(A_{x'},O)$ is a full rank matrix, hence we also have

$$\Pr(H(k,x)\oplus H(k,x')=y)\leq 2^m/2^{m+32}=2^{-32}.$$

Combining the above two cases, Item a) holds. The proof of Item b) is almost the same as that

of Item a). ■

Theorem 7 *If keystream bits generated by ZUC are indistinguishable from random, and the same nonce can never be re-used with the same secret key, then no forgery of a new message for 128-EIA3 can succeed with probability higher than* 2^{-32}.

Proof: An adversary queries a message M to the MAC algorithm and gets a tag T = H(k, M)⊕ z[L-1], where L= ⌈LENGTH(M)/32⌉+2, and LENGTH(M) denotes the bit length of the message M. He then tries to generate a valid tag T' for a new message M', i.e. T' = H(k, M')⊕ z[L'-1] where L'= ⌈LENGTH(M')/32⌉+2. We calculate the probability of success of the forgery as following.

1) If L'>L. z[L'-1] is a new word, indistinguishable from random, so the probability is 2^{-32}.

2) If L'=L. T'= H(k, M')⊕ z[L'-1] is equivalent to H(k, M)⊕ H(k, M') =T⊕ T'. The probability is no more than 2^{-32}, as h is 2^{-32}-AXU.

3) If L'<L. We notice that H(k, M')⊕ z[L'-1] = H(k, M'||1||0^s) where s = 31 if 32|LENGTH(M'), or s=63-(LENGTH(M') mod 32) if 32 is not a factor of LENGTH(M'). The probability is no more than 2^{-32}, as H(k, M'||1||0^s) is uniform. ■

12 Resistance against cryptanalytic attacks

12.1 Weak key attacks

First of all, it is worth to mention that the ZUC initialization process preserves the entropy of keys, i.e. there is a one-to-one mapping from the initial state of ZUC to the state after its initialization. It is observed by the evaluation report [33] that the mapping

$$(K, IV) \rightarrow \text{algorithm state immediately after initialization}$$

is injective. To see this, it suffices to show that the clocking rule during the initialization stage is bijective. So suppose that the state $(s_0,s_1,\ldots,s_{15},R_1,R_2)$ clocks to $(s'_0,s'_1,\ldots,s'_{15},R'_1,R'_2)$ and that X_0, X_1 and X_2 are the words derived from the former state. Given $(s'_0,s'_1,\ldots,s'_{15},R'_1,R'_2)$, one can

- compute $s_1,s_2,\ldots,s_{15}$ from $s'_0,s'_1,\ldots,s'_{14}$;
- compute X_0, X_1 and X_2 from $s_1,s_2,\ldots,s_{15}$;
- compute R_1 and R_2 from R'_1,R'_2, X_1 and X_2;
- compute the output W of the nonlinear function F from X_0, R_1 and R_2;
- compute s_0 from s_1, $s_2,\ldots,s_{15},s'_{15}$ and W:

$$s'_{15} = 2^{15}s_{15}+2^{17}s_{13}+2^{21}s_{10}+2^{20}s_4+257s_0+(w>>1) \bmod (2^{31}-1).$$

Note that gcd(257, 2^{31}-1)=1, the above equation on s_0 has exactly one solution. So $(s_0,s_1,\ldots,s_{15},R_1,R_2)$ is uniquely determined by $(s'_0,s'_1,\ldots,s'_{15},R'_1,R'_2)$. This shows that the clocking rule during ZUC initialization is bijective, and the mapping defined above is injective. This property helps to see the hardness of weak key attacks on ZUC.

For a given (K, IV)-pair, it is called a weak (K, IV)-pair if all cells s_i are equal to p after the ZUC initialization with K and IV, and the corresponding key K is called a weak key. As for the ZUC algorithm, it is very unlikely to exist such a weak state resulting from a (K, IV)-pair. This is because, when all cells s_i are equal to p in an initial working state, we view the unknown values of R_1 and R_2 of the nonlinear function F after ZUC's initialization as 64 binary variables, and track ZUC back to the initial state $(s_0,s_1,\ldots,s_{15},R_{1,0},R_{2,0})$. For any possible pair (R_1,R_2), since ZUC's initialization preserves key entropy, there must be an unique initial state $(s_0,s_1,\ldots,s_{15},R_{1,0},R_{2,0})$ that results in this (R_1,R_2) pair. Hence in this case each bit of the initial state $(s_0,s_1,\ldots,s_{15},R_{1,0},R_{2,0})$ can be uniquely represented as a function of the 64 binary variables of R_1 and R_2. Note that at the beginning of the initialization, $R_{1,0}$=0, $R_{2,0}$=0 and $s_i^{[8\text{-}22]}=D_i$ is a constant, where $s_i^{[8\text{-}22]}$ denotes all 15 bits of cell s_i from 8 to 22, i=0,1,…,15. It follows that we can establish 64+16×15=304 equations about the 64 binary variables of R_1 and R_2. Since the update of ZUC's states is a nonlinear process, it is roughly viewed that those equations are pairwise statistically independent. Since the number of such equations is far more than that of the binary variables, in general such an equation system is very unlikely to have a solution. Though it is beyond personal computational capability to ensure that there does not exist such a weak state (the time complexity to verify this is about $O(2^{64})$), it is believed that it is very unlikely that such a weak key K/IV exists.

12.2 Guess-and-Determine Attacks

The general idea of guess-and-determine attacks in [12]-[14] is: by guessing part of internal states of the target algorithm, combining with some known mathematical relations for the algorithm, to deduce the remaining unknown internal states.

ZUC appears to have strong resistance against guess-and-determine attacks. It is seen that the ZUC algorithm has 16×31+2×32=560 bits of internal states. Assume that an attacker tries to find these internal states at some time interval, and he tries to guess r bits of these states to determine the remaining 560-r states. Assume that the 560 bits of internal states have uniform probability distribution. Then the attacker needs at least $\lceil(560\text{-}r)/32\rceil$ key-words (the 32-bit words from the keystream) to establish algebraic equations, so as to possibly determine all the remaining unknown bits. For a successful guess-and-determine attack, it must be true that r<128. In this case the attacker needs at least 14 words of keystream. This means that the algebraic equations established using these 14 words will involve the values of R_1 and R_2 in F for at least 14 time intervals. Since the mechanism to update R_1 and R_2 is complicated and nonlinear, to deduce the values of the memory cells in the next time interval requires the current values of R_1 , R_2 and the inputs X_1 and X_2 which have to be guessed, and they are 128 bits all together. If the attacker tries to guess the values of the memory cells R_1 and R_2 in different time intervals, then the number of bits to be guessed is apparently no less than 128. This discussion only considers 2 consecutive time intervals, and in practice, since the real number of key-words needed to perform an attack is far larger than 2, it means that the number of internal bits to be guessed is far larger than 128. In this sense the ZUC algorithm has good resistance against the guess-and-determine attack.

12.3 BDD Attacks (from evaluation report [32])

An attack based on Binary Decision trees has been proposed by Krause [15], and it has a very low data complexity. In view of the large state size (560 bits), a straightforward application of this attack has no chance to succeed. We have not been able to identify a way that would optimize the attack so that it would approach the 2^{128} limit. We confirm that a BDD attack is no better than exhaustive search.

12.4 Inversion Attacks (from evaluation report [32])

The applicability of the inversion [16] and generalized inversion [17] attacks on filter generators are evaluated. As the two outer values s_0 and s_{15} are both used to produce the keystream, these attacks do not apply to ZUC.

12.5 Linear Distinguishing Attacks

The basic idea of linear distinguishing attacks [18]-[20] is first to establish a probabilistic linear relationship between the output keystream and the input sequence from the LFSR. Then, using the fixed linear iteration formula of the LFSR sequence, to obtain a linear iteration formula for the keystream that holds with certain probability, and if the probability appears to be obviously different from 1/2, then the output keystream can be distinguished from a random sequence.

As for the ZUC algorithm, we first consider the linear approximation of 2-round F. At two consecutive times, t and t+1, we have

$$(X_{0,t} \oplus R_{1,t}) \boxplus R_{2,t} = W_t \tag{23}$$

$$(X_{0,t+1} \oplus R_{1,t+1}) \boxplus R_{2,t+1} = W_{t+1} \tag{24}$$

$$W_1 = R_{1,t} \boxplus X_{1,t} \tag{25}$$

$$W_2 = R_{2,t} \oplus X_{2,t} \tag{26}$$

$$R_{1,t+1} = S(L_1(W_{1\mathrm{L}} \| W_{2\mathrm{H}})) \tag{27}$$

$$R_{2,t+1} = S(L_2(W_{2\mathrm{L}} \| W_{1\mathrm{H}})) \tag{28}$$

In the above, the nonlinear function F includes the S-box S and modulo 2^{32} addition $\boxplus$. One method to solve a system of nonlinear equations is to introduce new intermediate variables $R_{1,t}$, $R_{2,t}$, $R_{1,t+1}$, $R_{2,t+1}$, W_1 and W_2. Now we estimate the linear approximation of equations (23), (24), (25), (27) and (28), in which there are 3 modulo 2^{32} additions, 2 of S-box operations, see Figure 12.1.

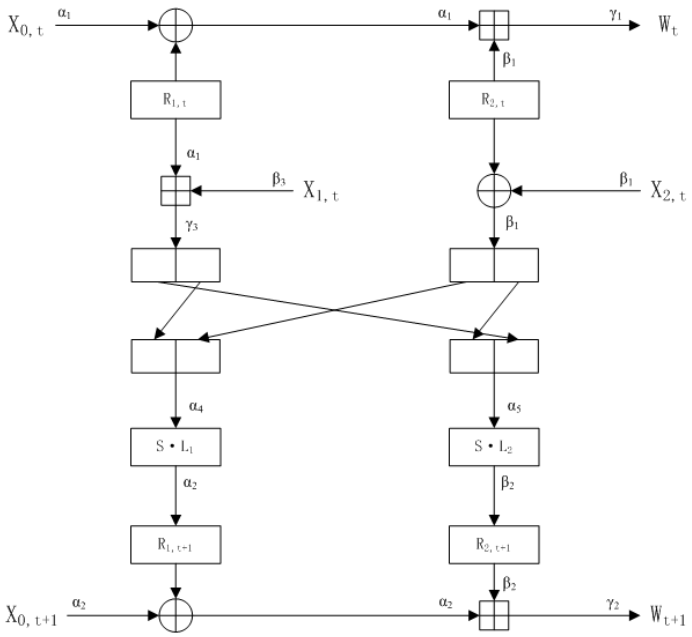

Figure 12.1: The linear approximation of two-round *F*

From Equation (23) we have:

$$\alpha_1 \cdot (X_{0,t} \oplus R_{1,t}) \oplus \beta_1 \cdot R_{2,t} = \gamma_1 \cdot W_t.$$

From Equation (24) we have:

$$\alpha_2 \cdot (X_{0,t+1} \oplus R_{1,t+1}) \oplus \beta_2 \cdot R_{2,t+1} = \gamma_2 \cdot W_{t+1}.$$

From Equation (25) we have:

$$\gamma_3 \cdot W_1 = \alpha_1 \cdot R_{1,t} \oplus \beta_3 \cdot X_{1,t}$$

From Equation (27) we have:

$$\alpha_2 \cdot R_{1,t+1} = \alpha_4 \cdot (W_{1L} \| W_{2H})$$

From Equation (28) we have:

$$\beta_2 \cdot R_{2,t+1} = \alpha_5 \cdot (W_{2L} \| W_{1H})$$

From the above process of linear approximation, it can be seen that, the intermediate variables $R_{1,t}$, $R_{2,t}$, $R_{1,t+1}$, $R_{2,t+1}$, W_1 and W_2 can be eliminated if and only if $\beta_1 = \alpha_{4L} \| \alpha_{5H}$ and $\gamma_3 = \alpha_{5L} \| \alpha_{4H}$ hold simultaneously. So we have the relation between the output variables W_t, W_{t+1} and the input variables $X_{0,t}$, $X_{0,t+1}$, $X_{1,t}$ and $X_{2,t}$:

$$\alpha_1 \cdot X_{0,t} \oplus \alpha_2 \cdot X_{0,t+1} \oplus \beta_3 \cdot X_{1,t} \oplus \beta_1 \cdot X_{2,t} = \gamma_1 \cdot W_t \oplus \gamma_2 \cdot W_{t+1} \qquad (29)$$

In our experiments, we searched the case when there is only one active S-box, and the best advantage from the linear approximations of Equation (29) is $\epsilon_F = 2^{-22.6}$, where the values of

$\alpha_1,\alpha_2,\beta_1,\beta_3,\gamma_1$ and γ_2 are shown in Table 12.1 (The values in Table 12.1 are taken from the evaluation report [33]).

Table 12.1: The values of $\alpha_1,\alpha_2,\beta_1,\beta_3,\gamma_1$ and γ_2 of one of the best linear approximations

Advantage	α_1	α_2	β_1	β_3	γ_1	γ_2
$2^{-22.6}$	01040405	00300000	01010405	01860607	01040607	00200000

In the following we will construct a (probabilistic) linear iteration relationship of keystream $\{Z_t\}_{t\geq 0}$ using the above linear approximation from two aspects:

From sections 10.1.1 and 10.1.3 it can be seen that, the coordinate sequences of an LFSR sequence $\underline{a}$ over the prime field GF(p) have large distance m ($\approx 2^{493}$) for shift-equivalence and large linear complexity d ($\approx 2^{495}$). These linear relationships can be used to eliminate the LFSR state variables in the linear approximation (29) to get a linear approximation of $\{Z_t\}_{t\geq 0}$, which requires at least 2^{493} bits of keystream, which is practically intractable.

Another approach is to find a multiple of $f(x)$ with low degree such that the number of its nonzero terms is as small as possible and the weights of its nonzero coefficients are always one, and then establish linear approximations of the LFSR using such a multiple, finally combine those linear approximations with those of 2-round F, i.e., equation (29), and establish a distinguisher only depending on the keystream. In the worst case if a trinomial multiple with low degree can be found (since the degree of any binomial multiple of $f(x)$ must be a multiple of $(p^{16}-1)/(p-1)$, thus its degree is at least $O(2^{465})$ and too large), under the assumption that the LFSR is linear over GF(2), when the best linear approximation (29) is used to construct a distinguisher, the advantage is about $2^{3-1}\epsilon_F^3 \approx 2^{-65.8}$. The number of keystream bits used by the above distinguisher is about $2^{131.6}$. If the effect of linear hulls is considered, the required number of keystream bits may be reduced somewhat, but the complexity of the attack will still be greater than the exhaustive key search. However, it should be pointed out that in the above worst case we ignored the difference between mod p and mod 2, which in fact is quite significant. Hence it seems extremely unlikely that a linear distinguishing attack with practical complexity is feasible.

12.6 Algebraic Attacks

The general idea of algebraic attacks [21]-[24] is to treat the whole encryption algorithm as an over-defined system of algebraic equations. Then the initial key or all the internal states at certain time interval can be recovered by solving this over-defined system of multivariate algebraic equations using some traditional methods, say linearization, re-linearization, Gröbner basis, XL method, F_4 and F_5 methods.

In the ZUC algorithm, since the feedback for the LFSR sequences determined by the characteristic polynomial (10) is nonlinear at bit level, the algebraic equations established using (10) at bit level will have algebraic degree at least 2 over GF(2). In the following, we show how a system of algebraic equations can be established based on the modulo p addition of 2 integers.

Let $x,y,z \in J$, $z=x+y \bmod p$, $x=x_{30}x_{29}\ldots x_1x_0$, $y=y_{30}y_{29}\ldots y_1y_0$, $z=z_{30}z_{29}\ldots z_1z_0$. Let c_{i+1} denote the carry when the i-th bits are summed, and under the modulo p operation we must have $c_0=c_{31}$. Then we have

$$z_i = x_i \oplus y_i \oplus c_i$$

$$c_{i+1} = x_i y_i \oplus (x_i \oplus y_i)\, c_i$$

From the above we can get

$$x_{i+1} \oplus y_{i+1} \oplus z_{i+1} = x_i y_i \oplus (x_i \oplus y_i)(x_i \oplus y_i \oplus z_i).$$

The above are in fact some algebraic equations of degree 2 for the corresponding bits of x,y,z. Similarly, we can establish algebraic equations for the case of modulo p addition of k variables. In general, when k is small, the algebraic equations for the k inputs $\boldsymbol{x_1},\boldsymbol{x_2},\ldots,\boldsymbol{x_k}$ and the output $\boldsymbol{y}$ will have algebraic degree k, here $\boldsymbol{y}=\boldsymbol{x_1}+\boldsymbol{x_2}+\ldots+\boldsymbol{x_k} \bmod p$. For the ZUC algorithm, since the feedback of the LFSR involves the modulo p addition of 6 terms, the algebraic equations for the states $s_{15},s_{13},s_{10},s_4,s_0$ and the feedback output s_{16} are very likely to have algebraic degree 6.

In section 10.3.1, we discussed some properties of the S-boxes. The 2 S-boxes in the nonlinear function F are algebraic immune of order 2, hence quadratic equations can be established for their inputs and outputs. Moreover, since the modulo 2^{32} addition $\boxplus$ will remain the linear relationship on the least significant bits, the other bits satisfy quadratic equations. If the intermediate variable W_1 is introduced, then the whole nonlinear function F can be represented by a system of algebraic equations of algebraic degree no more than 2 for the input variables X_0, X_1, X_2, the output W, the memory cells R_1, R_2 and the intermediate variable W_1. If no intermediate variables are introduced, then an equivalent system of algebraic equations for the input variables X_0, X_1, X_2, the output variable W and the memory cells R_1,R_2 can be established, but the algebraic equations are of algebraic degree 3.

The ZUC algorithm has a total number of $31\times16+2\times32=560$ internal state bits, and it is reasonable to assume that these bits are in random distribution at a certain time interval. Note that the algorithms outputs a 32-bit word at every clock pulse, hence in order to establish a over-defined system of algebraic equations that have a unique solution, it needs at least $18=\lceil 560/32 \rceil$ key-words. In the following we discuss a few ways to establish algebraic equations when 18 key-words are used:

1. **To eliminate all the intermediate variables introduced during the LFSR feedback**

   From the above discussion we know that, the LFSR feedback can induce some algebraic equations of degree 6 for s_{16} and $s_{15},s_{13},s_{10},s_4,s_0$, and the nonlinear function F can be used to establish a system of algebraic equations with degree 3. In addition, since X_1 will be affected by s_{16} after 5 clock pulses, the process to eliminate s_{16} from the current system of equations will result in the increase of the algebraic degree of the whole system, more precisely it will become a system of algebraic equations of algebraic degree 8. In this case the total number of variables needed in solving such equations is 1648 ($=16\times31+18\times2\times32$).

2. **To keep all the intermediate states introduced in the LFSR feedback as new variables**

   When all the intermediate states for the LFSR feedback computation are treated as new variables, since the algebraic equations for the LFSR states that can be established are of degree 6, hence the degree of the whole system of the algebraic equations are of degree 6. Under the condition that 18 key-words in total are used, then the total number of variables in such a system of algebraic equations is 2175 ($=16\times31+17\times31+18\times2\times32$).

3. To establish quadratic equations using the intermediate variables

We now consider to introduce new variables for the intermediate states to establish a system of algebraic equations of degree 2. For the computation of the LFSR feedback, we consider the following process:

$$y_1=(1+2^8)s_0 \bmod p,$$

$$y_2=2^{20}s_4+y_1 \bmod p,$$

$$y_3=2^{21}s_{10}+y_2 \bmod p,$$

$$y_4=2^{17}s_{13}+y_3 \bmod p,$$

$$s_{16}=2^{15}s_{15}+y_4 \bmod p.$$

It is easy to see that every of the above equations can induce 93 linearly independent quadratic equations, hence for the LFSR feedback computation, if we introduce intermediate variables y_1, y_2, y_3 and y_4, then we can establish 465 linearly independent quadratic equations for all the states and all the intermediate variables. In addition, if we introduce variable W_1 in the nonlinear function F, then it is also possible to establish a system of quadratic equations for the whole nonlinear function F with respect to the input variables X_0, X_1, X_2, the output variable W, the memory cells R_1, R_2 and the intermediate variable W_1. Note that the $\boxplus$ operation can establish 93 linearly independent quadratic equations involving 95 variables (another equivalent method is to establish 183 linearly independent quadratic equations involving 96 variables [32]), and the S-boxes S_0 and S_1 can establish 11 and 39 linearly independent quadratic equations respectively, therefore the whole ZUC algorithm can form a system of quadratic equations. When an attacker obtains 18 key-words, the total number of variables in the system is

$$16\times31+2\times32-1+17(5\times31+3\times32-2)=4792$$

and the number of linearly independent quadratic equations is

$$93+17(93\times5+2\times93+39+11)=12010.$$

The precise complexity of solving the above three algebraic systems is unknown, but using Proposition 6 of [25], it follows that an XL-like approach on this system is very unlikely to be faster than an exhaustive search for the key.

Note that the above discussion assumes that 18 key-words are used, this is the minimum number of key-words that can establish an over-defined system of algebraic equations. However in practice, when the linearization method is used to solve an algebraic equation system, the number of independent equations that can be established is likely to be far smaller than that of the variables, and hence the number of key-words needed will have to be larger than 18. This increased number of key-words will inevitably increase the number of intermediate variables, and the practical complexity is far higher than the above theoretical result.

12.7 Chosen IV Attacks

Chosen IV/Key attacks target at the initialization stage of stream ciphers. For a good stream cipher, after the initialization, each bit of the IV/Key should contribute to each bit of the internal states, and any difference of the IV/Key will result in an almost-uniform and

unpredictable difference of the internal states. Hence, any difference of the IV/Key will result in almost-uniform and indistinguishable difference of the output keystreams.

The initialization process of the ZUC algorithm has 32 iterations. We will consider the effect of any changes that the IV/Key has on the internal states after each of the iterations, i.e., when a specific difference of the IV/Key is chosen, how the difference propagates in each round of iteration.

After some computing test and analysis, we got the followings:

(1) When the 3-rd byte of ΔIV (the difference of IV's) is chosen to be non-zero (i.e., ΔIV[3] ≠ 0) and the remaining 15 bytes of ΔIV are all 0, the differences in both of the LFSR and the two memory cells R_1 and R_2 propagate slowest. In the following, we give a detailed analysis of this case.

Let ΔIV[3]=a≠ 0, then the differences of the 16 cells of the LFSR before the 1 round iteration are:

$$(s_{15},s_{14},s_{13},s_{12},s_{11},s_{10},s_9,s_8,s_7,s_6,s_5,s_4,s_3,s_2,s_1,s_0)=(0,0,0,0,0,0,0,0,0,0,0,0,a,0,0,0).$$

Then 32 rounds of iteration will take place. Table 12.2 gives the differences of the 16 cells of the LFSR after i rounds of iteration (i=1,2,…,21). It is easy to see that s_0=0 after 18 rounds, which means that the LFSR state is not random even after 18 rounds of iterations. By the cryptographic properties of the S-boxes and the linear transforms in the nonlinear function F, we know that b and c are not very random. However states marked with asterisk (*) in the low left corner of Table 12.2 indeed have good randomness (where the states on the left have better randomness than those on the right). Hence, we believe that, after 32 rounds of iteration, the differences of the 16 cells in the LFSR will be fairly even and unpredictable.

If we look at the difference variation of R_1 and R_2, it is noted that before the first iteration, R_1 and R_2 are both set to be zero. A simple deduction will see that after 1-7 iterations, the differences of both R_1 and R_2 are still zero. However after the 8-th round of iteration, although the difference of R_1 is still zero, that of R_2 is nonzero. Since the update of R_1 and R_2 involves S-boxes and the linear transforms, after the 9-th round of iteration, the differences of R_1 and R_2 have better randomness with the increase of the number of rounds of iteration. From this behavior it is believed that, after 32 rounds of iteration, the differences of R_1 and R_2 will be fairly random and unpredictable.

Table 12.2: The differences of the LFSR in each round of iteration

round	s_{15}	s_{14}	s_{13}	s_{12}	s_{11}	s_{10}	s_9	s_8	s_7	s_6	s_5	s_4	s_3	s_2	s_1	s_0
0	0	0	0	0	0	0	0	0	0	0	0	0	*a*	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	*a*	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*a*	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*a*
4	*b*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	*c*	*b*	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	*	*c*	*b*	0	0	0	0	0	0	0	0	0	0	0	0	0
7	*	*	*c*	*b*	0	0	0	0	0	0	0	0	0	0	0	0
8	*	*	*	*c*	*b*	0	0	0	0	0	0	0	0	0	0	0
9	*	*	*	*	*c*	*b*	0	0	0	0	0	0	0	0	0	0
10	*	*	*	*	*	*c*	*b*	0	0	0	0	0	0	0	0	0
11	*	*	*	*	*	*	*c*	*b*	0	0	0	0	0	0	0	0
12	*	*	*	*	*	*	*	*c*	*b*	0	0	0	0	0	0	0
13	*	*	*	*	*	*	*	*	*c*	*b*	0	0	0	0	0	0
14	*	*	*	*	*	*	*	*	*	*c*	*b*	0	0	0	0	0
15	*	*	*	*	*	*	*	*	*	*	*c*	*b*	0	0	0	0

16	*	*	*	*	*	*	*	*	*	*	*	*c*	*b*	0	0	0
17	*	*	*	*	*	*	*	*	*	*	*	*	*c*	*b*	0	0
18	*	*	*	*	*	*	*	*	*	*	*	*	*	*c*	*b*	0
19	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*c*	*b*
20	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*c*
21	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*

(2) When the 3-rd byte of ΔKey is chosen to be non-zero (i.e., ΔKey[3]≠ 0) and the remaining 15 bytes of ΔKey are all 0, the differences in R_1 and R_2 propagate slowest. We have that the difference in R_1 is zero after 8 rounds.

(3) When the 8-th byte of ΔKey is chosen to be non-zero (i.e., ΔKey[8]≠ 0) and the remaining 15 bytes of ΔKey are all 0, the difference in LFSR propagates slowest. We have that s_0=0 after 19 rounds.

Considering two kinds of extensions: (a) when we begin with the 1-st round, or the 2-nd round, or the 3-rd round, and backtrack in the reverse order, the steps for a good difference propagation could be added by 1-3 rounds. However it is unlikely to increase the number of rounds by 4 or more with nonrandom difference propagation; (b) when we consider that two or more bytes of IV/Key are active, the Hamming weight of the difference may be increased after several rounds of iteration. However, the two extensions are both limited, because of the mutual effect between the LFSR and F function. There are totally 32 rounds in the ZUC initialization, we believe that the 32-round iterations provide sufficient security against chosen IV/Key attacks.

12.8 Time-Memory-Data Trade-Off Attacks

12.8.1 Attacks against the function mapping internal state to keystream

The stream cipher comfortably resists traditional Time-Memory Tradeoff (TMTO) attacks against the internal state, because the state size is well over 2.5 times the secret key length (see [27]).

12.8.2 Attacks against the function mapping Key/IV pair to keystream

However, both external evaluation teams noted that the algorithm parameters used in LTE lend themselves to the alternative type of TMTO attack proposed by Hong and Sarkar in [29] and [30]. This is not a criticism of the ZUC cipher or the 128-EEA3 design — the comments apply equally to 128-EEA1 and 128-EEA2.

Apart from the 128-bit secret key, the other inputs to the encryption algorithm are a 32-bit COUNT, a 5-bit BEARER and a 1-bit DIRECTION. Collectively these act as the non-secret "Initialization Vector" (IV) for the cipher. Of these, only the 5-bit BEARER is at all unpredictable by an attacker intercepting encrypted messages – the COUNT is reset to 0 for each new secret key, and DIRECTION is uplink or downlink.

Hong and Sarkar describe generic Time-Memory Tradeoff (TMTO) attacks against stream ciphers, with the following form:

- the attacker precomputes keystreams for many Key/IV pairs;
- the attacker intercepts a large number of keystreams generated using many different secret keys[1];

[1] It's optimistic to assume that an attacker can intercept keystreams, rather than just encrypted versions of unknown messages; but this is the standard attack assumption when we are trying to design a cipher that will be secure irrespective of what's being encrypted.

- the attacker's goal is to recover the secret key used for anyone of those keystreams.

We measure the complexity of an attack by the one-off pre-computation time 2^p, the "online" computing time 2^t for the actual attack instance, the computer memory requirement 2^m, and the number 2^d of keystreams that the attacker has intercepted (from different keys – we assume that the first keystream sequence, for COUNT=0, is intercepted). Let the secret key length be k = 128, and let the number of unpredictable bits of IV be v = 5. Attacks are possible as follows:

- **[Babbage-Golic tradeoff]** By computing and storing a table of 2^m key/IV pairs and the keystreams they generate, the attacker can probably determine the key and used to generate one of the intercepted keystreams, if $m+d \geq k+v$. For instance, a table of 2^{93} (key/IV, keystream) values will be enough if 2^{40} keystreams are intercepted. The table can be computed once, and then used for repeated attack attempts on different sets of intercepted keystreams.
- **[Biryukov-Shamir tradeoff]** By precomputing 2^p key/IV pairs, and storing a table of size 2^m, an attack is possible with online time complexity t as long as $t \geq 2d$, $p+d \geq k+v$, and $m+t \geq k+v$. For example, with 2^{40} keystreams, an attack is possible with one-off precomputation time 2^{93}, and online time complexity 2^{80}, but computer memory requirement only 2^{53}.

Increasing the number of unpredictable IV bits increases the complexities of these attacks.

These observations were fed back to 3GPP SA3. Note that the attack model is somewhat questionable: it assumes that the attacker "wins" if she can recover the key for just one of the many keystreams she has intercepted. The complexity of recovering the key for any *particular* keystream is not improved significantly by these attacks, compared to generic key search techniques.

Nevertheless, there would be some advantages in increasing the number of unpredictable IV bits, if a source of them were readily available. All of the 128-EEA1/2/3 algorithms could easily be adapted to accommodate longer IVs.

13 Conclusion of the evaluation

The evaluation assessed many different classes of attacks and concluded that none were likely to succeed. A few components of the algorithm were identified for which the initially stated design rationale was not completely clear; further discussion with the designers has addressed those points, and the fuller explanations are now included in this design and evaluation report.

One stated objective for the design was that the new algorithms be substantially different from the first and second LTE algorithm sets, in such a way that an attack on any one algorithm set would be unlikely to lead to an attack on either of the others. In SAGE's view this objective is not fully met – there are some architectural similarities between ZUC and SNOW 3G, and it is possible that a major advance in cryptanalysis might affect them both. However:

- there are important differences too, so ZUC and SNOW 3G by no means "stand or fall together";
- and in any case the raison d'être of this new algorithm set is very different from that of the first two, so the objective is considerably less important than making the first and second algorithm sets different from each other.

SAGE therefore does not consider this a barrier to acceptance of the new algorithms. Indeed, both of the paid evaluation teams noted that the ZUC design inherits some strong security properties from SNOW 3G, while adding further protection against as yet unknown attacks.

The public evaluation on the June 2010 versions of the algorithm did bring to light some important problems with ZUC and with the integrity algorithm construction. We have seen other algorithms proposed in the literature that follow a dispiriting pattern: an attack is found, so a new version of the algorithm is published that blocks that attack, but then another rather similar attack is found … and sometimes this goes through several cycles before the designers finally acknowledge (or don't) that it was a fundamentally flawed design to begin with.

SAGE does not believe that ZUC is in that position. The problems found in the June 2010 version are thoroughly understood, and the changes made in the new version address the *causes* of the attacks. So we do not expect similar attacks to recur against the new version.

It is also true that a lot of the earlier analysis remains valid for the Jan 2011 versions. Nevertheless, the changes made are not trivial. The SAGE task force has recommended that the Jan 2011 version of the algorithms undergo a further period of public evaluation before being accepted into the mobile telephony standards.

14 Acknowledgements

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Annex A - External references

[4] A. H. Chan and R. Games, On the linear span of binary sequences from finite geometries, q odd, in Advances in Cryptology: Proceedings of Crypto 1986, Springer-Verlag, Berlin, 1987, 405-417.

[5] Iwata, T. & Kurosawa, K. OMAC: One-Key CBC MAC. *Fast Software Encryption 2003, Springer-Verlag,* 2003*, LNCS 2887*, 129-153.

[6] Bellare, M. New Proofs for NMAC and HMAC: Security Without Collision-Resistance. *Advances in Cryptology - CRYPTO 2006, Springer-Verlag,* 2006*, LNCS 4117*, 602-619.

[7] NIST Special Publication 800-38D. Recommendation for Block. Cipher Modes of Operation: Galois/Counter Mode (GCM) and GMAC. November, 2007.

[8] Carter, J. L., Wegman, M. N. Universal Classes of Hash Functions. *Journal of Computer and System Sciences,* 1979*, 18*, 143-154.

[9] Shoup, V. On Fast and Provably Secure Message Authentication Based on Universal Hashing *Advances in Cryptology -- CRYPTO 1996, Springer-Verlag,* 1996*, LNCS 1109*, 313-328.

[10] Krawczyk, H. LFSR-based hashing and authentication. Crypto'94. *Springer-Verlag,* 1994*, LNCS 839*, 313-328.

[11] Krawczyk, H. New hash functions for message authentication. Crypto'95. *Springer-Verlag,* 1995*, LNCS*, 129-139.

[12] P. Hawkes and G. Rose, Guess and determine attacks on SNOW, In Selected Area of Cryptography--SAC2002, LNCS 2595, pp.37-46, 2002.

[13] C.D. Canniere, Guess and determine attacks on SNOW, NESSIE Public Document, NES/DOC/KUL/WP5/011/a, 2001.

[14] H. Ahmadi, T. Eghlidos and S. Khazaei, Improved guess and determine Attack on SOSEMANUK, Tehran, Iran, 2006, http://www.ecrypt.eu.org/stream/sosemanukp3.html.

[15] M. Krause. BDD-Based Cryptanalysis of Keystream Generators. In L. Knudsen, editor, Advances in Cryptology - EUROCRYPT 2002, volume 2332 of Lecture Notes in Computer Science, pages 222–237. Springer-Verlag, 2002.

[16] J. Golic, A. Clark and E. Dawson. Inversion Attack and Branching. In J. Pieprzyk et al., editors, ACISP 1999, volume 1587 of Lecture Notes in Computer Science, pages 88–102. Springer-Verlag, 1999.

[17] J. Golic, A. Clark and E. Dawson. Generalized Inversion Attack on Nonlinear Filter Generators. IEEE Transactions on Computers, Vol. 49, No. 10, pages 1100–1109, October 2000.

[18] Dai Watanabe, Alex Biryukov,Christophe De Cannière. A distinguishing attack of SNOW 2.0 with linear masking method. In M.Matsui and R.Zuccherato eds. Selected

Areas in Cryptography 2003. Lecture Notes in Computer Science 3006. pp.222-233,2004. Springer-Verlag Berlin Heidelberg 2004.

[19] Don Coppersmith,Shai Halevi,Charanjit Jutla. Cryptanalysis of stream ciphers with linear masking. In M.Yung ed. CRYPTO 2002, LNCS 2442, pp.515-532,2002. Springer-Verlag Berlin Heidelberg 2002.

[20] Kaisa Nyberg,Johan Wallén. Improved Linear Distinguishers for SNOW 2.0. M.J.B. Robshaw ed. Fast Software Encryption 2006, Lecture Notes in Computer Science 4047, pp.144-162,2006. International association for cryptologic research 2006.

[21] Nicolas T. Courtois and Wili. Meier, Algebraic Attacks on Stream Ciphers with Linear Feedback, In Advances in Cryptology-EUROCRYPT 2003, LNCS 2656, pp. 346-359, Springer-Verlag, 2003.

[22] Sondre Ronjom, Tor Helleseth, Attacking the Filter Generator over GF(2^m), in Workshop Record of SASC 2007: The State of the Art of Stream Ciphers, eSTREAM report 2007/011 , 2007.

[23] WilliMeier, Enes Pasalic, and Claude Carlet, Algebraic attacks and decomposition of Boolean functions, In Advances in Cryptology-EUROCRYPT, 2004, LNCS 3027, Berlin: Springer-Verlag, 2004, pp. 474 - 491.

[24] Nicolas Courtois, Alexander Klimov, Jacques Patarin, and Adi Shamir. Efficient Algorithms for Solving Overdefined Systems of Multivariate Polynomial Equations.In Bart Preneel, editor, Advances in Cryptology - EUROCRYPT 2000, volume 1807of Lecture Notes in Computer Science, pages 392–407. Springer, 2000.

[25] Claus Diem. The XL-Algorithm and a Conjecture from Commutative Algebra. In Pil Joong Lee, editor, Advances in Cryptology - ASIACRYPT 2004, volume 3329 of Lecture Notes in Computer Science, pages 323–337. Springer, 2004.

[26] S. H. Babbage. Improved exhaustive search attacks on stream ciphers. In IEE European Convention on Security and Detection, volume 408, pages 161-165, 1995.

[27] S. H. Babbage and M. W. Dodd. The MICKEY Stream Ciphers. In New Stream Cipher Designs, volume 4986 of Lecture Notes in Computer Science, pages 191-209. Springer, 2008.

[28] A. Biryukov and A. Shamir. Cryptanalytic time/memory/data tradeoffs for stream ciphers. In Advances in Cryptology ASIACRYPT 2000, volume 1976 of Lecture Notes in Computer Science, pages 1-13. Springer, 2000.

[29] J. Hong and P. Sarkar. New Applications of Time Memory Data Tradeoffs. In Advances in Cryptology ASIACRYPT 2005, volume 3788 of Lecture Notes in Computer Science, pages 353-372. Springer, 2005.

[30] J. Hong and P. Sarkar. Rediscovery of Time Memory Tradeoffs. Cryptology ePrint Archive, Report 2005/090, 2005. http://eprint.iacr.org/.

[31] O. Dunkelman and N. Keller. Treatment of the initial value in Time-Memory-Data Tradeoff Attacks on Stream Ciphers. Information Processing Letters, 107(5):133-137, 2008.

[32] L.R. Knudsen, B.Preneel, V.Rijmen, Evaluation of ZUC, ABT Crypto, Version 1.1, 9 May 2010.

[33] C.Cid, S.Murphy, F.Piper, M.Dodd, ZUC Algorithm Evaluation Report, Codes & Ciphers Ltd., 7 May 2010.

[34] Xuanyong Zhu and Wenfeng Qi, On the distinctness of modular reductions of maximal length sequences modulo odd prime powers, MATHEMATICS OF COMPUTATION, Volume 77, Number 263, pp.1623–1637, July 2008.

[35] Bing Sun, Xuehai Tang and Chao Li, Preliminary Cryptanalysis Results of ZUC, appear in the First International Workshop on ZUC Algorithm, 12, 2010.

[36] Hongjun Wu, Cryptanalysis of the Stream Cipher ZUC in the 3GPP Confidentiality & Integrity Algorithms 128-EEA3 & 128-EIA3, appear at the sump session in ASIACRYPT 2010.

[37] Thomas Fuhr, Henri Gilbert, Jean-René Reinhard, and Marion Videau, A forgery attack on the candidate LTE integrity algorithm 128-EIA3, http://eprint.iacr.org/2010/618.